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# THE MONIST

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## RELATIVITY AND ITS PHILOSOPHIC IMPLICATIONS.*

*Aeltestes bewahrt mit Treue,*
*Freundlich aufgefasstes Neue.*
—*Goethe.*

OF all the natural sciences physiology would seem to touch the common interests of mankind most broadly and closely. As it becomes more and more rigorously scientific, it tends to resolve itself into chemistry. Witness the recent experiments of Loeb and Northrop on the aseptic flies of Bogolanow and Guyénot, indicating that the natural duration of an organism's life is the time necessary to complete a chemical reaction or a series of such reactions, and that it is doubled or trebled by lowering the temperature 10° C. Similarly, as chemistry becomes more strictly scientific, it tends to resolve into physics, and physics tends likewise to pass over into mechanics, which, we all know, lapses into mathematics. Mechanics has been defined by Kirchhoff, with startling clearness and exactness, thus: "Mechanics is the science of motion. Its problem is to describe the motions that take place in nature, to describe them completely and in the simplest way." A description is complete when it answers every rational question that can be put, but to say when it is simplest is not so easy. The simplest description possible to-day may not be the simplest possible to-morrow. Anyway, the final problem of all the physical sciences is thus seen to be *the description of the motions of the physical universe.*

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*Address delivered at the New Orleans meeting of the Southern Society of Psychology and Philosophy, April, 1920

Motion is change of place, or position in space, during time. To describe it is to tell how position changes as time goes on. Any point or position in our space is determined and distinguished from every other by three mutually independent measures expressed by numbers. Thus, if you know how far, that is how *many* feet, the point of this pencil is from the end-wall, from the side wall, and from the floor, then you know precisely where it is and where no other point is. Similarly, to know where a star is, you must know just three such things: as, its angular distances from due east and from the horizon—these two will enable you to point at it—and lastly, its distance in the direction thus determined. Three such things you *must know,* and anything else independent of these three you *cannot* know, about a position in our space. Since *three* such measurements are necessary and enough to fix any point in our space, we say that space is *three-dimensional.* Any three such measurements for a point are called its *coördinates.* We are all familiar with such, as latitude and longitude, which *two* fix a position on the earth-surface, which surface is accordingly called *two*-dimensional. We use such coördinates in addressing a letter, as 74 Ninth Street. Here 74 and 9 are the two coördinates fixing the place on the surface. If further we specify tenth floor, the 10 is the third coördinate, fixing the position in three-dimensional space.

All sorts of coördinates are possible and in actual use. The simplest are the so-called rectangular Cartesian (from their inventor, Descartes), the first already mentioned, as the three distances of a point from three walls or planes concurrent in a point called the origin, the starting-point, and denoted by the letter O. These three coördinate lengths are conveniently measured on the three edges of the room, meeting in the origin O; these edges are straight, lines at right angles, each to the other two, and are called the rectangular axes of X, Y, Z; the three lengths or coördinates are also denoted by ex, y, z; thus, the pencil-point, e. g., may be the point (5, 9, 12). Every such set of three values represents a point, and conversely, every point represents a set of three such values.

Each of us carries a set of such axes in his own body; the up-and-down axis, the right-and-left axis—formed by the out-stretched arms, and the fore-and-aft axis (a horizontal line, which we may imagine passing directly through the breast and back, midway of the shoulders). We tell where a point is by telling how high or low it is, how far to the right—or left, how far to the front or behind, up, right and front being marked +, down, left, and behind marked —. As we move hither and thither, we change the position of these axes, and this change expressed in mathematical terms is called *transformation of coördinates.* Any point whatever may serve as Origin O, and in space there is a threefold infinity of such possible origins; every hither-thither motion is a change of origin. A man may also wheel round, remaining upright; here the origin remains fixed, but the two horizontal axes are rotated round the third or vertical axis. Similarly he may rotate on a horizontal bar round the right-and-left axis, or even by a special mechanical contrivance round the fore-and-aft axis, and in each case through any angle whatever. Here again there is then a threefold infinity of possible changes of axes, making in all a six-fold infinity of possible transformations of coördinates. Any single transformation involving both any change of origin and any rotation of axis may be expressed by a set of three simple equations. The totality of all such possible transformation is called a *Group,* that is an aggregate of operations, such that the product of any two is itself some third member of the Group.

The stress here laid upon these notions has been intentional though reluctant; for they cannot be stressed too heavily. Indeed, the very heart and soul of the New Mechanics is this same *Transformation of Coördinates.*

Passing now to our subject-matter proper, we note that all men are familiar with the fact that motion is in some sense relative. You sit motionless in the car while it speeds along the track. You move with it but not in it; you are at rest relatively to the car but in motion relatively to the track and all outside space. A child knows all this but does not estimate the facts at their full significance. To bring them out clearly, let us imagine a number of coëxistent compunctual spaces. This need not bewilder anyone. You may take any rule and slip its edge along another rule-edge or any straight line. Here one straight line coincides with and slides along another. Now consider a surface. Suppose you pull on a perfect-fitting glove. Here the inside surface fits and moves on the surface of your hand; the two surfaces coincide point for point, and one moves on the other. Still better, since these surfaces are not isotropic, suppose a perfect sphere covered with a delicate film or gauze; then we may imagine this later slipping round over the sphere surface every way, the two always coincident point for point. Quite similarly we may imagine one isotropic space coinciding with another point for point and yet moving anyway in that other, always maintaining exact coincidence. So, too, we may imagine a third space moving in the second, a fourth in the third, and so on. We may imagine each of these spaces with its own proper color, red or green, blue or yellow, and all coexistent like all the colors in the white sunbeam. Or we may imagine a single point, when it moves, as staining each space with its track of a different color in each, red in one, green in another, and so on; and this seems to be very important.

Now imagine a man on a ship sailing west on the Equator $20^{m}$ an hour. He looks over the ship's side and drops a penny into the deep. It falls down straight, as he sees it—that is, it makes (say) a straight red track in the man's space, Sm. But not in the ocean space, So. To a man out there at rest on an island the penny falls to the sea on a green path, a parabola. Or consider simply the man. In his own ship-space, Sm, he is at rest. But in the Equator space, Sq, he is moving on a green path parallel to the Equator $20^m$ an hour west. Meantime, this Equator space is itself whirling eastward round the earth's axis in an axial space, Sx, at $1000^m$ an hour. In this Sx, then, the man is making a circular blue streak 980 miles an hour. Meantime this axis is being swept always parallel to itself round the sun on an immense ellipse at 18.6 miles per second. In this sun-space, Ss, then the man is tracing an enormous yellow spiral, one spire per day, moving round it 980 miles an hour and forward on it 18.6 miles a second. Meantime, the sun itself with its space is moving through universal space, Su, at a great rate, and in this Su the man is tracing out a purple path of almost indescribable complexity. Each and every one of these paths is just as actual as any other, each is relative to a higher space supposed at rest relative to the other space moving in it.

The question now arises, when does this stop? Where shall we find a space that is at rest absolutely, that is not swept along in some still higher space, so that with respect to this space any path will be the true path and any motion the true motion? You see at once that this sounds a little like asking for the true beginning of time and of the world. However, Newton and his successors believed firmly in an absolute space and an absolute time, though admitting of course the relativity of all observations so far. But could such an absolute space be actually found? The supposed answer was, Yes! For was not such an absolute actually present in the (assumed) universal aether which was (supposed to be) absolutely at rest? It was indeed the long and strenuous contention of George Gabriel Stokes that the ether was carried along by the planets in motion, much as the air is carried in a closed car; but profound researches especially of Lorentz, the great continuator of Maxwell, showed that this could not be, that electro-magnetic-optical measurements required that the aether be stationary, so that a body rushing through it would be like a man running through the still air—there would be a hurricane "Aether-wind" in the opposite direction. There then in the ether was supposed to be found the great unmoved immovable, to which all motions were to be finally referred.

A very natural question was, "How are we, how is the earth moving in this 'stagnant' ether?" In a letter directed to David Todd by Maxwell, March 19, 1879, shortly before his lamented death (November 5, 1879), and published in *Nature* shortly after (January 29, 1880), a method of astronomical attack upon the problem was indicated, and it was remarked that the time of passage of light forth and back between two points A and B would be longer if the points were in motion than if they were at rest; but inasmuch as the difference would depend on the reciprocal of the squared velocity of light, Maxwell thought it would be "quite too small" for possible observation. In this latter judgment he was at fault. In 1881 Albert Abraham Michelson, a young German, since become the especial boast of American science, performed at the Astrophysical Observatory at Potsdam a modification of Fizeau's famous experiment, by which the motion of the earth in the stagnant ether should have been distinctly revealed in the displacement of certain so-called interference fringes of yellow light. The ingenious experiment consisted in superposing two beams of light that had made equal trips forth and back from two mirrors, one of the trips in the direction of the earth's flight in its orbit, the other at right angles thereto. It was like comparing the time of swimming directly across a stream with that of swimming the same distance directly against, and then directly back with, the current. To the great surprise of the whole tribe of physicists, the experiment, though executed most successfully and with exceeding care, did not show any motion of the earth relatively to the ether. Some doubt, however, remained, for Michelson had supposed that the time of the light's transverse passage would be "entirely unaffected" by the earth's orbital motion; on the contrary, it would be lengthened by almost exactly half as much as would the time of passage along the earth's orbit. In fact if *l* be the length from the source, S, to the first mirror, M, v the velocity of the earth, *c* the velocity of light in vacuo (always and everywhere the same), T the time of passage to the mirror, $T_1$, the time of reflection back to S, then in the forward flash the light overtakes the mirror, with the effective speed of $c - v$, hence $T = \frac{l}{c - v}$; but in reflection the light meets the mirror, with the effective speed of $c + v$, hence the time $T_1 = \frac{l}{c + v}$; the sum of the two is the whole time or $T + T_1 = \frac{l}{c - v} + \frac{l}{c + v} = \frac{2\,l\,c}{c^2 - v^2}$, and the total distance covered is $\frac{2\,l\,c^2}{c^2 - v^2}$ or $\frac{2\,l}{1 - \frac{v^2}{c^2}}$. In the transverse passage the distance each way, forth and back, is the hypotenuse of a right triangle, its length is $l/\sqrt{1 - \frac{v^2}{c^2}}$ and the total time $2\,T^1$ is not $\frac{2\,l}{c}$ (as if the earth were at rest) but

$$\frac{2\,l}{c} \Big/ \sqrt{1 - \frac{v^2}{c^2}} = \frac{2\,l}{c} \left\{ 1 + \frac{1}{2} \cdot \frac{v^2}{c^2} + \frac{3}{8} \cdot \frac{v^4}{c^4} + \cdots\cdots\cdots\cdots \right\}$$

$$\text{whereas } T + T_1 = \frac{2\,l}{c} \cdot \frac{1}{1 - \frac{v^2}{c^2}} = \frac{2\,l}{c} \left\{ 1 + \frac{v^2}{c^2} + \frac{v^4}{c^4} + \cdots\cdots\cdots \right\}$$

In this case the earth-speed *v* is only $\frac{1}{10{,}000}$ of the light-speed C, hence $\frac{v^2}{c^2} = 10^{-8}$, and $\frac{v^4}{c^4} = -10^{-16}$.

This latter is far too small for any observation, hence it is neglected, and the difference of the two times is almost exactly $\frac{1}{c}\frac{v^2}{c^2}$ (just half of Michelson's first reckoning). After Lorentz had called attention to this oversight in a penetrating memoir (1886), Michelson along with Ed. W. Morley repeated the experiment in 1887 with far higher degree of accuracy but with the same negative result, and the latter again in 1905 with D. B. Miller with still greater care and precision, and still with the same result. In 1904 Trouton and Noble tested the matter electrically, by swinging up a very delicate parallel-plate condenser, the plates in a vertical plane free to rotate about a vertical axis. If the earth had any velocity relative to ether, a measurable swing should be set up in the condenser by reversing its charge regularly twice in the natural period of swing, just as one may set a pendulum to swinging by pushing it ever so lightly, this way and that, at each instant of its swing through its mid-position. Again it was the unexpected that happened: the condenser would not swing, thereby indicating there was no detectable motion of the earth relatively to the ether.

These and other indications pointing the same way are now accepted as conclusive. So far as we can observe the earth is absolutely at rest in ether; in spite of its spin round its axis, its orbital dance round the sun, its age-long flight among the stars, it does not move relatively to ether.

Of course, thinkers came forward with suggestions to explain this confounding fact. First, it seems, the British Fitzgerald advanced an idea, which met the fate of so many British thoughts, and was past unheeded until presented afterwards independently by Lorentz of Amsterdam—namely, that of the two supposedly equal arms of Michelson's apparatus, the one lying orbitally was shortened by the velocity of the earth's motion; in fact, that every body shrinks in the line of its motion in the fraction $\sqrt{1-\frac{v^2}{c^2}}$, but not transversely, at all. This would account exactly for the absence of the expected displacement; but such a shortening could not be detected by measuring the two arms, for if laid down side by side they would be found equal, since each would shrink in the same ratio. In fact, no changes in size that affect the measure and the measured in the same way can be detected by comparison. If space and everything in it should suddenly or gradually expand a million-fold, or shrink down to a centillionth, or alternately dilate and contract in any measure whatever, we could not discern it by any conceivable measurement.

Lorentz' hypothesis, which found great favor, was proposed in 1895. In 1904 he expressed it mathematically as a transformation of coördinates, in a memoir on "Electromagnetic phenomena in a system moving with any velocity smaller than that of light." He assumed two sets of axes, one at rest called X, Y, Z; another (X′, Y′, Z′) with the sames axes at first but with the origin moving out along the *X*-axis with the velocity *v* for any time, *t*. Plainly then the new coördinates of any point (x′, y′, z′) would be according to the Galilei-Newtonian Physics $x' = x - vt$, $y' = y$, $z' = z$, and $t' = t$, since clearly each x would be shortened by the length *vt* while y and z and t would be unchanged. But Lorentz showed that on his new contraction-hypothesis, there would result a very different set of equations, viz:

$X' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$, $Y' = Y$, $Z' = Z$, $T' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$, and these equations constitute the famous *Lorentz-Transformation.*

The next year (1905) Einstein arrived at the same transformation, in ignorance of Lorentz's work, and along quite another line of thought. He did not think of the length along the line of motion as compressed by the motion, somewhat as an accordeon moving in the air, but as a necessary result of the *relativity of motion.* In fact, he laid aside entirely the notion of the stagnant immovable ether, assuming that there was no final system of reference at all, that one system was just as good as any other, if only it moved uniformly on a right line relatively to the other, and he laid at the basis of his whole construction these two postulates which together constitute the so-called Principle of Special (or restricted) Relativity:

> 1. The laws according to which the conditions of physical systems vary are the same when referred to *any* two systems of coördinates that are moving relatively to each other uniformly and on a straight line.
>
> 2. In a coördinate system at rest, light (in vacuo) moves always and everywhere with the same velocity *c,* no matter whether the source of light be in motion or at rest.

These two postulates Einstein then pursued into all of their logical consequences. He found they yielded the Lorentz-transformation and the negative result of Michelson and cleared up other dark points. It resulted that time no less than length and space was relative. This astonishing fact comes to light in many ways, as thus: Suppose there are two points A and B on a straight track, and let the midpoint M between them be exactly determined. Now suppose two flashes, one from A, one from B, are perceived as one by an observer at rest at M; then for him the two flashes are (psychologically) simultaneous. But now suppose at the instant of flash an observer carrying his coördinate system with him, or on a moving train, was gliding through M towards B with half the velocity of light then he would meet the flash from B 1/3 of the way from M to B, and the flash from A would catch up with him just as he reached B. He would not perceive the flashes as simultaneous, they would be separated by an interval of time; this interval would vary with the speed of the observer. If this observer moved with the speed of light, he would meet the flash from B halfway and he would not be overtaken at all by the flash from A; he would not see it, for him it would not exist. Hence it appears that simultaneity is only relative; that just as a point is describing variously shaped and colored paths in the various coëxistent or coördinate spaces or coördinate systems, so the same event is happening at all sorts of times in these various spaces or coördinate systems. Time has meaning not at all in itself but only in the various spaces or coördinate-systems, and has different meanings in different systems.

This fundamental transformation of Lorentz and Einstein discloses various astonishing relations. The shortening of all lengths in the line of motion has already been mentioned. If we inquire into the times we easily discover that time is similarly affected. Suppose a clock beating seconds in the origin O′ of the moving system X′ Y′ Z′. To an observer in the unmoving origin O of the system x y z the clock will not beat seconds but each interval will be slightly longer, viz, one second divided by the fraction $\sqrt{1-\frac{v^2}{c^2}}$ Again, we are all familiar enough with adding velocities. If you walk forward four miles an hour in a car going $30^m$ an hour, your total forward speed is reckoned at $34^m$ an hour; if you walk back it is reckoned at $26^m$ an hour. But this simple addition does not hold in the new Physics. There the formula is no longer $W = u + v$, but $W = \frac{u+v}{1+\frac{uv}{c^2}}$, where u and v are the velocities added and W is their sum. Now suppose u and v each equal to c, the velocity of light. Then we get $W = \frac{c+c}{1+\frac{c^2}{c^2}} = \frac{2c}{2} = c$. That is, the sum of two velocities each equal to the velocity of light is still only the velocity of light. If from a gun moving with the speed of light a ball were discharged with the speed of light, as observed from without, it would not leave the gun. No additions to the speed of light can increase that speed. No speed greater than the speed of light is observable. If a body, as this rule, were to start straight up at a speed of 161,000 miles a second, its length would become only half, for the value of the ratio $\sqrt{1 - \frac{v^2}{c^2}}$ would be ½. If its speed were to increase up to c, the speed of light, its length would cease to be, it could no longer be observed. Thus it appears that this speed c plays the part of infinity, it cannot be exceeded—which suggests the most heart-searching suspicions.

You are ready to say, "Oh! these things are not so, they merely seem so. The length seems shorter because it is moving so fast, the clock runs slower, because it takes some time to flash back the sign of its finger on the clock-face, the body ceases to be when moving away with the speed of light, because any flash issuing from it is borne away as fast as it approaches us and so can never reach us." This objection seems to be just, but it does not touch the nerve of the matter. For these seemings, these appearances, are after all the only evidences or signs that we have of any physical existence. The doctrine of relativity says we must take these apparent times and spaces at their face values; we can do nothing else, for we have no absolute time or space with which to compare them. As they seem so they are; there is no court of higher jurisdiction to which we can appeal.

So much for the Special-Relativity theory. It is concerned solely with what we might call inertial or Galileian realms, as the inter-stellar spaces where Galilei's law of inertia holds, and every body is at rest or moving uniformly on a right line. You have observed that not a word has been said thus far of any force, or mass, or attraction, or acceleration, or rotation (except in speaking of possible changes of axes). But all these things are matters of experience and must be taken into account. The result therefore will be the Universal-Relativity theory. Before attacking this a word or two concerning certain profound thoughts of Hermann Minkowski, the most brilliant of recent analysts. They were expressed in a wonderful address at the 80th Meeting of the Society of German Physicists and Physicians at Cologne, September 21, 1908. Einstein has appropriated these thoughts and avows that but for them his own ideas would never have left their cradle. It had long been known that in the general Equations of Transformation there was a so-called Double Invariance: the forms of the equations of motions were not changed by any change of coördinate axes, the equations held just as well for one set of axes as for another, and secondly the forms were not changed by any uniform motion of the axes, but held as well when the axes were moving uniformly as when they were at rest. It was Minkowski's great achievement to bring out the full meaning especially of this latter invariance. He begins with the obvious fact that time and space are known in experience only together, never apart. All that happens, happens somewhere and somewhen. Also, in the equations of motion the time enters like the coördinates x, y. z. Accordingly, any point in space at any time is supposed by Minkowski to be specified not by three coördinates (x, y, z) but by four (x, y, z, t), t, of course, meaning any time-value. It is not merely a point *there* but a point *there-then,* and is called a *World-point.* The whole life, the whole career, of such a point would be the whole assemblage of all its possible theres and thens, or better, its *there-thens,* and would be a line not in space simply nor in time simply, but in space-time in a world not of 3 but of 4 dimensions. Such a line is a *World-line.* The whole of Space-Time, the whole of world-history, consists of the totality of such world-points and world-lines. But as we could not observe a single point in space or time, any single world-line would also escape us, and our observations must be wholly of the intersections of world-lines, which make up for us the whole of experience, the totality of world-events. Thus is formed a single static arrangement of all the possibilities of the universe around us, a simple four-fold picture of all that is or was or will be. In this 4-dimensional world-canvas this present meeting will appear as a highly complex knot of intersecting world-lines countless in number; any moment will appear as a cross-section of the gigantic knot. This section will not be a point, as when you cut a line, nor a line, as when you cut a surface, nor a surface, as when you cut a solid, but it will be a solid itself, with the two four-fold extents of space-time fitting upon it and separated by it. So the Conference at Paris appears as another much immenser and worse-tangled knot of world-lines in another world-region; so also the signing of the Declaration of Independence, the crossing of the Rubicon, and so on. The world-line of a point forever at rest would be a line parallel to the time-axis; of a point moving uniformly in a right line it would be a right line sloped to the time-axis. Now, however, since motion is entirely relative we may choose any point as the origin of our system of axes, and in this system at rest the point will be at rest. We ought then to be able to choose our time-axis so as to make any right-line we please parallel to it, and such is the case. In fact, by mathematical ingenuities Minkowski is able to preserve the fundamental invariances of the transformation of coördinates while placing time upon exactly the same footing as any other dimension, as length or breadth or depth. Accordingly he may then speak of before-and-after exactly as of right-and-left, up-and-down, back-and-forth, without ever making any distinction between time and space, which have thus become melted indistinguishably together. This may confound us at first, but only think that all these distinctions are purely relative. Now the ceiling is up, but on changing our origin to a higher floor the same ceiling would be down. Also by changing the origin and at the same time turning the axes the up-and-downs, right-and-lefts, back-and-forths, would all become completely fused, mixed up quite inextricably with each other. The eggs would be scrambled past all unscrambling, but the new axes would be just as good as the old; the scrambled eggs would combine into new eggs quite as pretty as the old ones. However, these new eggs would not be any better than the old. Precisely so, in Minkowski's four-fold time-space, there remains no longer any distinction of time-axis and space-axes; hence it becomes inept to speak any longer of x, y, z, and t- axes, since, on transformation t may no longer refer to time, which has become blended completely with space. Instead of X Y Z T we may better write $X_1$ $X_2$ $X_3$ $X_4$ which emphasizes the fact that the axes are quite alike, differing only in the order in which they are named. Now by change of origin and by rotation of axes any one set of axes may be transformed into any other set while still maintaining the same form for the equations of motion, still upholding the invariances that characterize the group of transformations.

Now what does this mean in ordinary speech? It means this: A man on earth and a man in the moon or on the dog-star Sirius would have entirely different sets of axes; up and down would not mean the same for the two, even as it does not mean the same in London and in Tokio, in New York and in New Orleans. Neither would sooner-and-later mean the same thing to the two men; and each might displace his axes and whirl them about as he pleased. Nevertheless, by virtue of the invariants of the group of transformations, the equations of motion describing the time-space world of each would be equivalent. The 4-dimensional geometries of the two would be the same, and the measurements that they made upon a common object would agree at least as required by the fundamental Lorentz-equations. What then has Minkowski accomplished? He has turned three-fold kinematics into four-fold statics, and mechanics into 4-dimensional geometry. The changing universe of time becomes a single instantaneous universal presentation in 4-dimensions, where past, present, and future appear all alike, all equally real, but none absolute. It is a kind of ecstatic revelation, a stereoscopic vision of eternity in 4-dimensions. We all know how the stereoscope constructs the plane surface of the picture into a three-fold space; somewhat similarly the thought of Minkowski projects all three-fold space along a fourth axis into this hyperstereoscopic vision of unending space and time.

Einstein returned upon his problem of relativity with a mind enriched by Minkowski's ideas, which Minkowski himself alas! did not live to elaborate. Thus far the framework or system of reference was supposed to be a so-called Galilean body, that is, a body rigid and at rest or moving uniformly in a right-line; and all such systems were equally good; the equations of motion were alike for all. Such was the principle of special relativity which declared that the physical description of nature-processes remains the same no matter which of any two systems, S and S′, be employed provided only it be rigid, and the relative motion of the two be uniform and rectilinear. There is *no* preferred system, such as the ether was long supposed to be, referred to which the motions would be true and real, while referred to any other they would be only apparent and unreal. On the contrary, all motions referred to all such Galileian systems are equally real and equally justified. Now when this postulate is combined with the second, namely, that light moves *in vacuo* always and everywhere with the same speed, then the old Galileian formulae of transformation turn into the new Lorentzian formulate, with all the remarkable results already noted, and many others.

But such passages from one Galileian or inertial body in uniform rectilinear motion to another such, are not the only ones imaginable, just as Galileian bodies are not the only ones imaginable. In fact, such bodies are as scarce as hens' teeth; they are never present in our experience but are only conceptual fictions (à la Vaihinger), though none the less important and even indispensable. But ordinary bodies vary endlessly in their motions. The question now arises: Is it possible to conceive and state the motions of nature so that the description may be equally correct no matter how the system of reference be moving? Such was Einstein's audacious thought, certainly one of the most daring in the history of the human mind. We may say his special doctrine of relativity gave a formal description of nature-motions equally correct whether viewed from the top of the Liberty Statue or from an airplane sailing straight across New York harbor, but now he sought an expression that should still be equally correct if the observer were looping the loop in his airplane or tossed violently in a storm-cloud.

The first impulse to this transcendent doctrine seems to have been given in a difficulty connected with the apparent increase of electro-magnetic inertia of a body in motion, without accompanying increase of weight, of its gravitational mass. These two have never been considered the same. At a certain point between earth and moon, comparatively near the latter, a cannonball's weight would be zero, since the pulls of earth and moon would balance, but the explosion necessary to shoot it off with a given speed would be the same as here on earth, for its inertial mass would be unchanged. However, observation ran to the effect that the ratio of the two masses for any body was unchanging and for all bodies the same. A wonderfully delicate experiment of Eotvös in 1891 seemed to show that the ratio could not vary so much as one twenty-millionth. It now occurred to Einstein that inertia and weight are the same thing under different conditions. The like has perhaps occurred to many; I myself have never been able to get rid of the suspicion; but Einstein assumed it boldly as the basis of a daring construction. He saw a man fall from a high building; luckily, the man was not seriously hurt, and he afterwards described to Einstein his sensations during the descent. We all know that actual weight, say at the Equator, is blended of earth-pull and so-called centrifugal force; if the rotation-velocity of the earth were 17 times as great as it is, it would completely annul the earth-pull, and the hugest stone would be lighter than a feather. Now suppose a large chamber in a Galilean field, that is, either at rest or moving uniformly and straight; let an angel or one of Maxwell's "demons" seize the chamber by a hook in the top and bear it aloft with constant pull, that is, with constant acceleration, with steadily increasing speed. How would the man feel? Lighter than a feather before, he is now heavy; he feels in feet and legs the upward push of the floor; he presses on it, and so does everything in the chamber; if he lets anything go, it drops to the floor and all things precisely alike. If he fastens a cord to the top and any body to the cord-end, the body hangs down, the cord is tense. In fact, for all possible observation he finds himself in a gravitation field, solely in virtue of his uniformly accelerated motion.

Of course, it has long been known that the straight path of a projectile (say) in a Galileian field becomes a curve (as a parabola) in a gravitation field; it remained for Einstein to apply the principle to light. Fifty years ago Maxwell showed that light must press longitudinally, along its rays, against the mirror it strikes; thirty years later this was verified experimentally by the Russian Lebédev and still later, in 1904, and still more accurately by the Americans, Nichols and Hull. But this pressure was due to the light's electromagnetic energy, which endows the light with inertia-mass; did it also have gravitational mass, that is, weight? According to Einstein's so-called *Principle of Equivalence* (of an accelerated Galileian field to a gravitational field) the light must have weight, and a star-ray must be bent sunward in passing near the edge of the sun, so that to an observer on earth it would appear displaced from the sun about 1″.74, just twice the calculated Newton-Maxwell displacement .87″. It is the striking confirmation of this prediction in the British observations on the total eclipse of May 29, 1919, that has so startled the world. But if the light is bent its speed must vary, contrary to Einstein's fundamental assumption; but this was made only for Galilean fields and does not hold for intense gravitation fields. Modifications must be continually introduced into growing scientific structures. For the simpler case, earlier treated, is later perceived to be only a limiting extreme case of a more general and complicated one—into which it is merged not as a contradiction but as a particular under a universal, and it can hope for no kindlier fate.

So much for the uniform gravitation field equivalent to the uniformly accelerated Galileian field. But here on earth the gravity field is not uniform; a body's weight varies from point to point, it is less on the equator, greater towards the poles. So the solution of the problem is only started; how are varying gravitation fields to be treated? Still another difficulty before trying to solve this. We have seen that the actual gravitation field, as at the equator, is a blend of gravitation and rotation. Consider now how this rotation must affect time-space measures. Suppose a man on a disc that rotates steadily like a merry-go-round. To himself he is at rest but pulled towards the rim; to an outside Galileian observer he is in uniform circular motion. Let the man set down two similar clocks, one on the rim, one at the centre. To the Galileian the moveless central clock runs faster than that on the rim. So also to the disc-man, who would count himself at rest wherever he stood, his clocks would vary in rate from point to point, and time would mean nothing definite, and neither would the phrase "at the same time."

Now let him measure lengths on his disc, and let the Galileian observe him. To this latter the foot-rule at the centre is shortened when laid tangentially on the rim in motion, but not when laid on the diameter of the disc, at right angles to the motion. Hence the rim turns out longer than in a circle at rest, and the calculated value of the perimetric ratio $\pi$ would be greater than our familiar 3.14159 ...... So then the Euclidian geometry would not apply, the rotation would expand the circumference leaving the diameter unchanged.

If such be the effect of rotation with its so-called centrifugal force exactly the opposite must be that of centripetal acceleration in a gravity-field, as when a mass particle is at the centre of the disc: all radii being in the direction of the varying motion would be shortened in the varying fraction $\sqrt{1-\frac{v^2}{c^2}}$, while all circumference or tangential elements would remain unchanged; hence the unit used in measuring the circumference would be shortened in measuring the diameter, so that the ratio would fall out less than 3.14159....., the circumference would be in effect contracted as regards the diameter, an extremely important fact to be noted hereafter.

The fundamental insistent question now recurs: Can we find a space, a scaffolding, or set of coördinates that shall be related to this world of rotations and central accelerations as the rectangular axial system is related to the Galileian inertial world? In this latter all rectangular systems are alike justified, the description of an event in any one is quite as correct as in any other. In our actual centripetal-centrifugal world are there any corresponding systems, all alike valid for the description of events? That is, in which the equations of motion do not change form on transforming coördinates from system to system? The answer is, yes! They are highly generalized coördinates called Gaussian, from the Göttingen prince of mathematicians, Carl Friedrich Gauss. We all know so-called coördinate paper. Imagine a sheet with the lines as close as you please, each line numbered; each point on the page is fixt as the intersection of two such lines, U and V. Think now of a curved surface covered with such a double system of lines, parallels of latitude and meridians of longitude, similarly numbered and named, each line of each system intersecting each of the other, but none of its own system. Now suppose the paper or the sphere-surface distorted in any way. The double system of lines will be deformed but will determine the points of the surface precisely as before, the point (5, 7) will remain the point (5, 7). The two essential conditions are maintained, namely: Each line of each system intersects each line of the other system in one and only one point, and by making the distance as between two points P and P′ small at will the differences du and dv of the corresponding coördinates may be made small at will.

What has been said of surfaces holds just as well of 3- or 4- or n-dimensional extents; the Gaussians define points in all just the same way. Think of this room as filled with fine spider-threads, 3 systems, u, v, w, from wall to wall; up and down, east and west, north and south—so that through each point in the room there shall pass one and only one line of each system and in each system let all the lines be numbered exactly. Also suppose the room filled with jelly, to keep each line in place. Now let a September tornado strike the room and twist the jelly mass into any shape but not tear it at any point. Then the lines will be contorted every way, but their intersections remain definite, the point (3, 7, 9) will still be the point (3, 7, 9). Now expand your imagination from the room to all space, with the same systems of lines or Gaussian coördinates, $u$, $v$, $w$. And now suppose a fourth system added, the time-system $t$, giving a four-fold determination in the four-fold time-space world. Then you have the coördinate system used by Einstein. In its use he employs the most powerful and refined mathematical methods of the Italian analysts Ricci and Levi-Civitá, and his stupendous problem is to find such a four-fold system of Gaussian coördinates as shall enable us to represent in equations the complex positions, velocities, and accelerations at any and all instants and on transformations to any other four-fold Gaussian system shall continue to represent them with the same exactness. How he solves the problem by the agency of invariants and covariants and contravariants and fundamental tensors and other such enginery of analysis cannot be set forth now and here; suffice it, that by means of covariance in a certain quadratic differential of four terms, and by means of tensors, that is, expressions such that when a linear relation connects tensors of the same character in one system of coördinates it connects them in all systems, he arrives at laws and equations that do not contradict the Newtonian but include them as special extreme and limiting cases. Thus, Einstein's two equations for the motion of a particle in a plane gravitational field agree with Newton's, except in the addition in one of them of the extremely small term $2\,m\frac{h^2}{r^3}$. But this happens to be just enough to explain numerically the 43″ per hundred years of rotation of the orbit of Mercury, which Newton's equations left unexplained.

Other examples could be given, but it is not in this higher arithmetical accuracy that the supreme interest lies. The important point is that the jelly-space through which the Gaussians run and always hold their own need not be Euclidian; in fact, the new physics does away with the notion of *force,* along with nearly the whole conceptual apparatus of the elder mechanics and describes the motions of nature as being what they are, and as simulating the operation of forces in Euclidian space, *by virtue of merely taking place in non-Euclidian space,* in space that is curved and crinkled like the surface of the sea—in fact, as it were in a jelly-space that has been twisted and strained but not rent by a tornado. No one balks at a crinkled sea-surface, but many balk at a crinkled sea-solid. The difference is not logical but psychological, in our depictive power. A three-fold space may be curved in a four-fold space quite as well as a two-fold in a three-fold, or a one-fold in a two-fold. So much admitted, and it must be, it then follows that the necessities for gravitative and doubtless other forces in the old physics lay in the hills and valleys of space itself—as Homer would say, "In the numberless folds of Olympus," that is, of the starry expanse.

It is not then that the particle at the sphere—or circle—centre attracts other particles around it, but this particle is a *fiction* (as Vaihinger would say) now to be supplanted by the superior fiction of a curvature in that particular region of space or rather time-space.

This all-important conception of matter as essentially a phenomenon of irregularity in the curvature of space may at first blush revolt the reader, and it may be well to prepare his mind by reference to certain familiar analogous facts. Imagine a circle in its plane; it is smooth, it has the same curvature at every point; an experience confined within it would have no sense of resistance or urge or irregularity of motion. Similarly on a perfectly smooth sphere-surface, everywhere equally curved. But now suppose the curve not exactly circular but set with minute unequal aufractuosities here and there along its whole length; then in such an experience confined to such a deformed circle a sense of resistance, urge, or unequal motion might well be present. Similarly, if the sphere-surface were not smooth but indented irregularly like the rind of an orange, an experience wholly therein might well involve such ideas as effort, restraint, and varying motion; more accurately, such ideas or experiences might well be interpreted or symbolized under the form of such varying curvature. Now on passing to three-dimensional space the analogy holds perfectly. We must remember that the curvature of such a space is definable and expressible quite as precisely as the curvature of a curve or a surface; and that our three-dimensional space must or may be thought as a border between two four-fold extents, just as the circle is a border between two plane surfaces, its inside and its outside, and the sphere-surface as a border between two solids, inside and outside; also that the plane touches the circle in and out along its whole length, and that solid space touches the sphere-surface, in and out, at every point. Likewise we must think of four-fold space as touching our (three-fold) space at every point and the curvature-irregularities of the latter (its matter-points or regions) as jutting into four-fold space this way and that, forming dents and jags (sharp curvatures) much as the bends, highths, and hollows on the circular curve and the sphere-surface. In such a three-fold space, rugged as a mountain-range, all the phenomena of molar and molecular mechanics, of force, mass, energy, and the like, might well present themselves in experience as characteristic of pure motion.

But the reader will still insist that motion implies *something that moves,* and that this ever persistent something is precisely what he calls *matter,* and is not any flexure of space in which the motion takes place. This objection borrows all its vesture of extreme plausibility from the fundamental assumption of the "natural man" that the reality of the physical world around us consists of *objects* (Kant's "things in themselves") that are what they are independently of mind, soul, or spirit, which merely perceives them and pries into their properties as best it can. Now it is exactly this *Thing-ism* that Einstein's Relativism refutes and expels from Philosophy. Even his doctrine of Special relativity accomplishes this, for it shows that the length (and therewith every dimension of a body) belongs to it not in itself but only as a relation of observer-and-observed and may even vanish under certain conditions of motion. The general theory of relativity takes another long step forward in resolving *thing-character* into *space-property,* in particular, into curvature.

Nevertheless, the reader will remain unsatisfied until he takes the last decisive step towards a positivistic idealism and definitely recognizes that the objective character of the *mover* is only a construct or objectification of psychic activity, of spirit itself. It is not enough to say with Kant that the subject imposes its own conditions or forms of time and space upon the material of sensation freely offered it in experience. This still leaves untouched the independent objective datum, the main source of the woes of philosophy; besides, it is not possible to assign any satisfactory meaning to this imposition of time and space (as forms of intuition) upon the stuff of experience. Nay; the *objects* in question are not the *discoveries* but the *creations* of psychic activity, they are not merely objects but also *objectifications,* for the basic law of spirit-activity is to project, to construct, to image, to *objectify.* Time and space and the categories are forms or ways in which this continuous externalization takes place, as indeed, in a more refined sense, are all science, all art, and even all history. The varying curvature of space, which gives rise to the notion of permanent objective independent matter, is the image, construct, or objectification of the diversified manifold nature of spirit (or psychic activity) itself, and the conservation of mass and energy is the construct or symbol of abiding unity amid such unending diversity. As Goethe has so impressively declared:

> And all the wide world's wild commotion
> Is endless rest in God the Lord.

To return from a long but important and even necessary digression, we have in fact seen how the centripetal acceleration works in effect a contraction of the circumference and a diminution of the ratio $\pi$. A similar change we actually know very well here on earth. From the pole as centre with a quarter-meridian as radius we may draw a great circle on the earth-surface, namely, the Equator. This latter is in length $\pi$ times the diameter measured *through* the earth, but it is only two times its own arc-diameter measured *on* the earth, through the pole. If we trace a small circle, a parallel of latitude, similarly, we find its ratio to its arc-diameter on the earth is still less than $\pi$, but approaches the value $\pi$ as the circle shrinks up closer and closer round the pole, and very near the pole the value would be indistinguishable (by measurement) from $\pi$. You say, "But the arc-diameter is not the real diameter." Very true, if you pass from the earth-surface of two dimensions to the interior earth-space of three. But if you know nothing of that interior three-fold space, and stick to the earth-surface, then your arc-diameter is the only diameter and it functions in your surface-space exactly as a straight line, and on this earth-surface the ratio of circumference to diameter would always be found less than $\pi$, but approaching $\pi$ for all very small circles. Now as our two-fold earth-surface is curved in three-fold star-space, so may our star-space be curved in four-fold space; and so may our four-fold space-time be curved in a conceivable five-fold space. Accordingly, there is nothing bizarre in supposing that gravitation around and towards the sun is only a dynamical way of stating the pure statical geometric fact that the sun is the name for a certain hump or ridge in space in whose neighborhood the curvature is marked and the circummetric ratio is less than 3.14159. . . . If a path were drawn around his hump l, as the orbit of Mercury, it would be shorter than the corresponding ellipse calculated by Newton and Leverrier in Euclidean space; if we take this ellipse, cut it at one point, straighten it out, and then bend it along on the orbit or path of Mercury round the hump, it will lap over very slightly at the end, and this slight excess of 43″ per century appears in the discrepancy for which the old mechanics could not account, but which the new mechanics explains without remainder.

There are various other considerations that urge upon us this conception of our space as curved and finite. Finite, but not bounded. This important distinction was clearly drawn by Riemann. A circle or any closed curve is finite but has no bounds. A caterpillar may crawl round forever on the rim of a small tub and may very properly fancy he is taking an endless but not an infinite journey. So a football surface is unbounded; an ant might hurry and skurry over it forever without stay or hindrance, and construct an admirable two-dimensional geometry; but the surface is not infinite, it contains just so many square feet, no more. Similarly our space may be boundless but yet finite, having just so many cubic miles.

Grave difficulties meet the supposition that our space is infinite. For if attractive matter were scattered throughout with any finite average density d, however small then the attraction at any point would be infinite. For describe around any point an enormous sphere of radius R. The attraction at the point would vary as the mass contained,—that is as $d\times R^3$,—divided by $R^2$; that is, it would vary as R, and would increase with R beyond all limit. This cannot be, hence the mass cannot be spread, however thinly, through infinite space. Hence it has been supposed that the starry universe, the world of matter, is a disc-like island in an infinite ocean of emptiness; and Seeliger especially has tried to modify the Newtonian law of inverse squares; but none of this satisfies. It seems better to assume the indicated spheroidal form of our space, to suppose it is Riemannian rather than Euclidean. Let us suppose that spider lines or light-rays issue from a point P in every direction. On each one measure off a distance R. The ends of all these measures will lie on a sphere of radius R about the center P. Let R increase. In Euclidean space the surface would grow larger and larger, always equally $4\pi R^2$. In Riemannian it would not, but would attain a certain maximum and then shrink down to a point. The surface of a sphere would be $4\pi R^2\overline{\sin X}^2$; and as this angle X increases from 0 up to 90°, the sin X rises from 0 to 1; as it increases from 90° to 180° the sin X shrinks from 1 to 0. Precisely so, if you describe circles about a pole on the earth-surface, the circle—length will be always $2\pi R\sin X$ (where X is the co-latitude); this sin X increases from 0 to 1 as X increases from 0 to 90°; there the circle is the Equator $2\pi R$; from this on, the circle shrinks as X increases from 90° to 180°, and finally shrinks to a point in the opposite pole. Exactly so, if our space be curved and Riemannian; its volume will be $2\pi^2 R^3$, and the value of R turns out to be simply related to the average density d of matter, namely: $R^2 = \frac{1.08\times 10^{25}}{d}$ metres. Naturally this squared radius varies inversely as the density of matter, for *presence of matter* is only a fictitious way of naming the *fact of curvature.*

There is then nothing outré or outlandish about this notion of curved finite space of 3-dimensions and of space-time of 4-dimensions; it is only a natural extension of the notion of a sphere-surface, of a curved finite space of 2-dimensions. But we need no more suppose that this finite space is exactly spherical and smooth-curved everywhere alike than that the earth is exactly spherical and perfectly smooth. On the contrary, the unequal and indeed irregular distribution of matter is only an aspect or symbol of the irregularity of curvature, which is itself the symbol or objectification of singularity inherent in the constructive soul. As the density of matter is never very great, so neither is the curvature anywhere very sharp; space spreads out like a mighty Pacific scarcely so much as fretted by a summer breeze. As Homer says, "Out from the current profound of the River unrippled of Ocean."

Even after leaving out the most that was first written down, the time is well-nigh out and little is left for discussing the "philosophic implications," the main theme originally in mind. Einstein himself never mentions or hints at any; he is obsest with measurements, with his rods and clocks and signals and his purely mathematical apparatus; but Max Planck, the father of the quantum, justly likens Einstein's revolutionary thought to that of Copernicus, as "surpassing in boldness everything previously suggested in speculative natural philosophy and even in philosophical theories of knowledge," in which judgement J. J. Thompson seems to concur. For my own part, I can hardly be a fair, even if I were a competent, judge, so completely and perfectly does the Einstein theory fit in with the general world-view that I have long cherished and am gradually shaping into expression.

For assuredly the rigorous and uncompromising doctrine of universal relativity leaves no escape from the conclusion that the subjects of its treatment are themselves the creatures of the intelligence that so boldly, so audaciously manipulates them, in whose hands they are more plastic than wax, being fused and compounded and contorted at will, to depict and reflect the purely analytic properties of a group of coördinate transformations. If the elements of the time-space world were themselves the inflexible constituents of a world self-existent and independent of the intelligence that was merely contemplating their play on the stage of the physical universe, then indeed all this metrical-mathematical prestidigitation of Einstein's would seem to be as insolent as it was meaningless and absurd. He would indeed seem to be more ridiculous than a dog baying at the moon. But the case is altogether another if the time-space world be itself the obedient creature of mind, the construct of intelligence itself, which has brought it into being as its own self-expression and which is pursuing its age-long, its eternal, task of reducing that construct to an order and system that shall be perfectly rational and self-consistent, that is, that shall symbolize in all its structural details the harmony, the diapason, of the universal soul. Accordingly, the Einstein theory would appear to be the latest and the longest step ever taken in the rational organization and interpretation of human experience. This experience, insofar as it is conscious, seems to begin with the most formless and undifferentiated sensations; it advances to ideas, to *Vorstellungen,* in the creation around us of the perceptual, particularly, of the visual world, and through this it moves on still higher and further to the fashioning of concepts, to the splendid architectonic of science and philosophy. As intuition to the world of percepts, so is theory to the world of concepts. It is in this third heaven of spiritual energy that Einstein and the lamented Minkowski have celebrated their marvelous triumph. But it must not be supposed that this triumph is by any means complete or final. There are not only many problems still unsolved but many that can hardly as yet be stated. However, the indication seems clear that the physical world must henceforth be regarded as a sacrament, as the outward sign of an inward grace, as the shining symbol of one all-pervasive spiritual activity. Not only then do all the physical sciences in turn merge into mathematics, but mathematics itself, their queen, is merged in logic, and logic is the return of psychology upon itself, and philosophy is the deep-eddying ocean-stream engirdling and engulfing them all.

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